APJ ABDUL KALAM TECHNOLOGICAL UNIVERSITY

B. Tech 2019 Regulations

Mathematics Minor

Curriculum & Syllabus

Minor in Mathematics - Basket of 5 courses

Curriculum

Sl.No.	Course Code	Course Name	Semester of Study
1	MAT281	Advanced Linear Algebra	S3
2	MAT282	Mathematical Optimization	S4
3	MAT381	Random Process and Queuing Theory	S5
4	MAT382	Algebra and Number Theory	S6
5	MAT481	Functional Analysis	S7

SEMESTER III

MINOR

CODE MAT 281	Advanced Linear Algebra	CATEGORY	L	T	P	CREDIT
		VAC	3	1	0	4

Preamble: This course introduces the concept of a vector space which is a unifying abstract frame work for studying linear operations involving diverse mathematical objects such as n-tuples, polynomials, matrices and functions. Students learn to operate within a vector and between vector spaces using the concepts of basis and linear transformations. The concept of inner product enables them to do approximations and orthogonal projects and with them solve various mathematical problems more efficiently.

Prerequisite: A basic course in matrix algebra**.**

Course Outcomes: After the completion of the course the student will be able to

CO 1	Identify many of familiar systems as vector spaces and operate with them using vector space tools such as basis and dimension.
CO 2	Understand linear transformations and manipulate them using their matrix representations.
CO 3	Understand the concept of real and complex inner product spaces and their applications in constructing approximations and orthogonal projections
CO 4	Compute eigen values and eigen vectors and use them to diagonalize matrices and simplify representation of linear transformations
CO 5	Apply the tools of vector spaces to decompose complex matrices into simpler components, find least square approximations, solution of systems of differential equations etc.

Mapping of course outcomes with program outcomes

	PO 1	PO 2	PO 3	PO 4	PO 5	PO 6	PO 7	PO 8	PO 9	PO 10	PO 11	PO 12
CO 1	3	3	3	3	2	1			1	2		2
CO 2	3	3	3	3	2	1			1	2		2
CO 3	3	3	3	3	2	1			1	2		2
CO 4	3	2	3	2	1	1			1	2		2
CO 5	3	3	3	3	2	1			1	2		2

Assessment Pattern

Bloom's Category	**Continuous Assessment Tests**		**End Semester Examination**
	1	**2**	
Remember	5	5	10
Understand	10	10	20
Apply	10	10	20
Analyse	10	10	20
Evaluate	15	15	30
Create			

End Semester Examination Pattern: There will be two parts; Part A and Part B. Part A contain 10 questions with 2 questions from each module, having 3 marks for each question. Students should answer all questions. Part B contains 2 questions from each module of which student should answer any one. Each question can have maximum 2 sub-divisions and carry 14 marks.

Course Level Assessment Questions

Course Outcome 1 (CO1):

1. Show that the $S_1 = \{(x, y, 0) \in R^3\}$ is a subspace of R^3 and $S_2 = \{(x, y, z) \in R^3 : x + y + z = 2\}$ is not a subspace of R^3
2. Let S_1 and S_2 be two subspaces of a finite dimensional vector space. Prove that $S_1 \cap S_2$ is also a subspace. Is $S_1 \cup S_2$ s subspace. Justify your answer.
3. Prove that the vectors $\{(1,1,2,4), (2,-1,5,2), (1,-1,-4,0), (2,1,1,6)\}$ are linearly independent
4. Find the null space of $A = \begin{bmatrix} 1 & 2 & 0 & -1 \\ 2 & 6 & -3 & -3 \\ 3 & 10 & -6 & -5 \end{bmatrix}$ and verify the rank nullity theorem for $m \times n$matrix in case of A

Course Outcome 2 (CO2)

1. Show that the transformation $T; R^2 \to R^3$defined by $T(x, y) = (x - y, x + y, y)$ is a linear transformation.
2. Determine the linear mapping $\varphi; R^2 \to R^3$ which maps the basis $(1,0,0), (0,1,0)$ and $(0,0,1)$to the vectors $(1,1), (2,3)$and$(-1,2)$. Hence find the image of $(1,2,0)$
3. Prove that the mapping $\varphi; R^3 \to R^3$ defined by $T(x, y, z) = (x + y, \quad y + z, z + x)$ is an isomorphism

Course Outcome 3(CO3):

1. Prove that the definition $f(u, v) = x_1y_1 - 2x_1y_2 + 5x_2y_2$for $u = (x_1, y_1)$ and $v = (x_2, y_2)$ is an inner product in R^2.
2. Prove the triangle inequality $||u + v|| \leq ||u|| + ||v||$ in any inner product space.
3. Find an orthonormal basis corresponding to the basis $\{1, tcost, sint\}$ of the subspace of the vector space of continuous functions with the inner product defined by $\int_0^\pi f(t)g(t)dt$ using Gram Schimdt process .

Course Outcome 4 (CO4):

1. Consider the transformation $T: R^2 \to R^2$ defined by $(x, y) = (x - y, 2x - y)$. Is T diagonalizable. Give reasons.

2. Use power method to find the dominant eigen value and corresponding eigen vector of $\begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 18 & -1 & -7 \end{bmatrix}$.

3. Prove that a square matrix A is invertible if and only if all of its eigen values are non-zero.

Course Outcome 5 (CO5):

1. Find a singular value decomposition of $\begin{bmatrix} 1 & -1 \\ -1 & 1 \\ 2 & -2 \end{bmatrix}$

2. Find the least square solution to the system of equations

$$x + 2y + z = 1, 3x - y = 2, 2x + y - z = 2, x + 2y + 2z = 1$$

3. Solve the system of equations $2x_1 + x_2 + x_3 = 2$,$x_1 + 3x_2 + 2x_3 = 2$, and $3x_1 + x_2 + 2x_3 = 2$ by LU decomposition method.

Syllabus

Module 1

Vector Spaces, Subspaces -Definition and Examples. Linear independence of vectors, Linear span, Bases and dimension, Co-ordinate representation of vectors. Row space, Column space and null space of a matrix

Module 2

Linear transformations between vector spaces, matrix representation of linear transformation, change of basis, Properties of linear transformations, Range space and Kernel of Linear transformation, Inverse transformations, Rank Nullity theorem, isomorphism

Module 3

Inner Product: Real and complex inner product spaces, properties of inner product, length and distance, Cauchy-Schwarz inequality, Orthogonality , Orthonormal basis, Gram Schmidt orthogonalization process. Orthogonal projection. Orthogonal subspaces, orthogonal compliment and direct sum representation.

Module 4

Eigen values, eigenvectors and eigen spaces of linear transformation and matrices, Properties of eigen values and eigen vectors, Diagonalization of matrices, orthogonal diagonalization of real symmetric matrices, representation of linear transformation by diagonal matrix, Power method for finding dominant eigen value,

Module 5

LU-decomposition of matrices, QR-decomposition, Singular value decomposition, Least squares solution of inconsistent linear systems, curve-fitting by least square method, solution of linear systems of differential equations by diagonalization

Text Books

1. Richard Bronson, Gabriel B. Costa, *Linear Algebra-an introduction,* 2nd edition, Academic press, 2007

2. Howard Anton, Chris Rorres, *Elementary linear algebra: Applications versio,* 9th edition, Wiley

References

1. Gilbert Strang, *Linear Algebra and It's Applications,* 4th edition, Cengage Learning, 2006
2. Seymour Lipschutz, Marc Lipson, *Schaum's outline of linear algebra*, 3rd Ed., Mc Graw Hill Edn.2017
3. David C Lay, *Linear algebra and its applications,3rd edition, Pearson*
4. Stephen Boyd, Lieven Vandenberghe, I*ntroduction to Applied Linear Algebra: Vectors, Matrices,and Least Squares,* Cambridge University Press, 2018
5. W. Keith Nicholson, *Linear Algebra with applications*, 4th edition, McGraw-Hill, 2002

Assignments:

Assignment should include specific problems highlighting the applications of the methods introduced in this course in science and engineering.

Course Contents and Lecture Schedule

No	**Topic**	**No. of Lectures**
1	**Vector spaces (9 hours)**	
1.1	Defining of vector spaces , example	2
1.2	Subspaces	1
1.3	Linear dependence, Basis , dimension	3
1.4	Row space, column space, rank of a matrix	2

1.5	Co ordinate representation	1
2	**Linear Mapping (9 hours)**	
2.1	General linear transformation, Matrix of transformation.	2
2.2	Kernel and range of a linear mapping	1
2.3	Properties of linear transformations,	2
2.4	Rank Nullity theorem.	1
2.5	Change of basis .	2
2.6	Isomorphism	1
3	**Inner product spaces (9 hours)**	
3.1	Inner Product: Real and complex inner product spaces,	2
3.2	Properties of inner product, length and distance	2
3.3	Triangular inequality, Cauchy-Schwarz inequality	1
3.4	Orthogonality, Orthogonal complement, Orthonormal bases,	1
3.5	Gram Schmidt orthogonalization process, orthogonal projection	2
3.6	Direct sum representation	1
4	**Eigen values and Eigen vectors (9 hours)**	
4.1	Eigen values and Eigen vectors of a linear transformation and matrix	2
4.2	Properties of Eigen values and Eigen vectors	1
4.3	Diagonalization., orthogonal diagonalization	4
4.4	Power method	1
4.5	Diagonalizable linear transformation	1
5	**Applications (9)**	
5.1	LU decomposition, QR Decomposition	2
5.2	Singular value decomposition	2
5.3	Least square solution	2
5.4	Curve fitting	1
5.5	Solving systems of differential equations.	2

SEMESTER IV

MINOR

CODE MAT 282	Mathematical optimization	CATEGORY	L	T	P	CREDIT
		VAC	3	1	0	4

Preamble: This course introduces basic theory and methods of optimization which have applications in all branches of engineering. Linear programming problems and various methods and algorithms for solving them are covered. Also introduced in this course are transportation and assignment problems and methods of solving them using the theory of linear optimization.Network analysis is applied for planning, scheduling, controlling, monitoring and coordinating large or complex projects involving many activities. The course also includes a selection of techniques for non-linear optimization

Prerequisite: A basic course in the solution of system of equations, basic knowledge on calculus.

Course Outcomes: After the completion of the course the student will be able to

CO 1	Formulate practical optimization problems as linear programming problems and solve them using graphical or simplex method.
CO 2	Understand the concept of duality in linear programming and use it to solve suitable problems more efficiently .
CO 3	Identify transportation and assignment problems and solve them by applying the theory of linear optimization
CO 4	Solve sequencing and scheduling problems and gain proficiency in the management of complex projects involving numerous activities using appropriate techniques.
CO 5	Develop skills in identifying and classifying non-linear optimization problems and solving them using appropriate methods.

Mapping of course outcomes with program outcomes

	PO 1	PO 2	PO 3	PO 4	PO 5	PO 6	PO 7	PO 8	PO 9	PO 10	PO 11	PO 12
CO 1	3	3	3	3	2	1			1	2		2
CO 2	3	3	3	3	2	1			1	2		2
CO 3	3	3	3	3	2	1			1	2		2
CO 4	3	2	3	2	1	1			1	2		2
CO 5	3	3	3	3	2	1			1	2		2

Assessment Pattern

Bloom's Category	**Continuous Assessment Tests**		**End Semester Examination**
	1	**2**	
Remember	5	5	10
Understand	10	10	20
Apply	10	10	20
Analyse	10	10	20
Evaluate	15	15	30
Create			

End Semester Examination Pattern: There will be two parts; Part A and Part B. Part A contain 10 questions with 2 questions from each module, having 3 marks for each question.

Students should answer all questions. Part B contains 2 questions from each module of which student should answer any one. Each question can have maximum 2 sub-divisions and carry 14 marks.

Course Level Assessment Questions

Course Outcome 1 (CO1):

1. Without sketching find the vertices of the possible solutions of $-x + y \leq 1$, $2x + y \leq 2$, $x,\ y \geq 0$

2. Solve the LPP $Max\ 8x_1 + 9x_2$ subject to $2x_1 + 3x_2\ \leq 50$, $3x_1 + x_2 \leq 3$,$x_1 + 3x_2 \leq 70$, $x_1, x_2 \geq 0$ by simplex method

3. Solve the LPP $Max\ - x_1 + 3x_2$ subject to $x_1 + 2x_2\ \geq 2$, $2x_1 + 6x_2 \leq 80$, $x_1 \leq 4$, $x_1, x_2 \geq 0$ by Big M method.

Course Outcome 2 (CO2)

1. Formulate the dual of the following problem and show that dual of the dual is the primal $Max\ 5x_1 + 6x_2$ subject to $x_1 + 9x_2\ \geq 60$, $2x_1 + 3x_2 \leq 45$, $x_1, x_2 \geq 0$
2. Using duality principle solve $Min\ 2x_1 + 9x_2 + x_3$ subject to $x_1 + 4x_2 + 2x_3\ \geq 5$, $3x_1 + x_2 + 2x_3\ \geq 4$, , $x_1, x_2 \geq 0$
3. Use dual simplex method to solve $Min\ z = x_1 + 2x_2 + 4x_3$ subject to $2x_1 + 3x_2 - 5x_3\ \leq 2$, $3x_1 - x_2 + 6x_3\ \geq 1$,$x_1 + x_2 + x_3\ \leq 3$, $x_1 \geq 0\ x_2 \leq 0$, x_3 unrestricted

Course Outcome 3(CO3):

1. Explain the steps involved in finding the initial basic solution feasible solution of a transportation problem by North West Corner rule..

2. A company has factories A, B and C which supply warehouses at W_1 , W_2 and W_3. Weekly factory capacities are 200, 160 and 90 units respectively. Weekly warehouse requirement are 180,120 and 150 respectively. Unit shipping cost in rupees is as follows

16	20	12
14	8	16
26	24	16

Determine the optimal distribution of this company to minimise the shipping cost

3. In a textile sales emporium, sales man A, B and C are available to handle W, X Y and Z. Each sales man can handle any counter . The service time in hours of each counter when manned by each sales man is as follows

	A	B	C	D
W	41	72	39	52
X	22	29	49	65
Y	27	39	60	51
Z	45	50	48	52

Course Outcome 4 (CO4):

1. Draw the network diagram to the following activities.

Activity	1-2	1-3	1-4	2-5	3-5	4-6	5-6
Time duration	2	4	3	1	6	5	7

2. The following table gives the activities in a construction project and other relevant information

Activity	1-2	1-3	1-4	2-5	3-5	4-6	5-6
Time duration	2	4	3	1	6	5	7

Find the free , total and independent float for each activity and determine the critical activities.

3. For a project given below find (i) the expected time for each activity (ii) T_E, T_L values of all events (iii) the critical path.

Task	A	B	C	D	E	F	G	H	I	J	K
Least time	4	5	8	2	4	7	8	4	3	5	6
Greatest time	6	9	12	6	10	15	16	8	7	11	12
Most likely time	5	7	10	4	7	8	12	6	5	8	9

Course Outcome 5 (CO5):

1. Consider the unconstrained optimization problem $max\ f(\boldsymbol{x}) = 2x_1x_2 + x_2 - {x_1}^2 - 2{x_2}^2$. Starting from the initial solution $(x_1, x_2) = (1,1)$ interactively apply gradient search procedure with $\in = 025$ to get an approximate solution.

2. Consider the following nonlinear programming problem.

$$Max\ f(\boldsymbol{x}) = \frac{1}{1+x_2} \text{ subject to } x_1 - x_2 \leq 2, x_1 \geq 0, .\ x_2 \geq 0$$

   Use KKT condition to show that$(x_1, x_2) = (4, 2)$ is not an optimal solution

3. Minimize $f = -4x_1 + x_1^2 - 2x_1x_2 + 2x_2^2$ subject to $2\,x_1 + x_2 \leq 6,\ x_1 - 4\,x_2 \leq 0, x_1 \geq 0,\ x_2 \geq 0$ using Quadratic programming method.

Syllabus

MODULE I

Linear Programming – 1 : Convex set and Linear Programming Problem – Mathematical Formulation of LPP, Basic feasible solutions, Graphical solution of LPP, Canonical form of LPP, Standard form of LPP, slack variables and Surplus variables, Simplex Method, Artificial variables in LPP, Big-M method.

MODULE II

Linear Programming – 2 : Two-phase method, Degeneracy and unbounded solutions of LPP, Duality of LPP, Solution of LPP using principle of duality, Dual Simplex Method.

MODULE III

Transportation and assignment problems: Transportation Problem, Balanced Transportation Problem, unbalanced Transportation problem. Finding basic feasible solutions – Northwest corner rule, least cost method, Vogel's approximation method. MODI method. Assignment problem, Formulation of assignment problem, Hungarian method for optimal solution, Solution of unbalanced problem. Travelling salesman problem

MODULE IV

Sequencing and Scheduling : Introduction, Problem of Sequencing, the problem of n jobs and two machines, problem of m jobs and m machines, Scheduling Project management-Critical path method (CPM), Project evaluation and review technique (PERT), Optimum scheduling by CPM, Linear programming model for CPM and PERT.

MODULE V

Non Linear Programming: Examples nonlinear programming problems- graphical illustration**.** One variable unconstrained optimization, multiple variable unconstrained optimization- gradient search**.** The Karush –Kuhn Tucker condition for constraint optimization-convex function and concave function. Quadratic programming-modified simplex method-restricted entry rule, Separable programming.

Text Book

1. Frederick S Hillier, Gerald J. Lieberman, Introduction to Operations Research, Seventh Edition, McGraw-Hill Higher Education, 1967.
2. Kanti Swarup, P. K. Gupta, Man Mohan, Operations Research, Sultan Chand & Sons, New Delhi, 2008.

Reference

1. Singiresu S Rao, Engineering Optimization: Theory and Practice ,New Age International Publishers, 1996
2. H A Taha, Operations research : An introduction , Macmillon Publishing company,1976
3. B. S. Goel, S. K. Mittal, Operations research, Pragati Prakashan, 1980
4. S.D Sharma, "Operation Research", Kedar Nath and RamNath - Meerut , 2008.
5. Phillips, Solberg Ravindran ,Operations Research: Principles and Practice, Wiley,2007

Assignments:

Assignment should include specific problems highlighting the applications of the methods introduced in this course in science and engineering.

Course Contents and Lecture Schedule

No	**Topic**	**No. of Lectures**
1	**Linear programming – I (9 hours)**	
1.1	Convex set and Linear Programming Problem – Mathematical Formulation of LPP	2
1.2	Basic feasible solutions, Graphical solution of LPP	2
1.3	Canonical form of LPP, Standard form of LPP, slack variables and Surplus variables, Artificial variables in LPP	1
1.4	Simplex Method	2
1.5	Big-M method.	2
2	**Linear programming – II (9 hours)**	
2.1	Two-phase method	2
2.2	Degeneracy and unbounded solutions of LPP	2
2.4	Duality of LPP	1
2.5	Solution of LPP using principle of duality	2

2.3	Dual Simplex Method.	2
3	**Transportation and assignment problems - (9 hours)**	
3.1	Balanced transportation problem	2
3.2	unbalanced Transportation problem	1
3.3	Finding basic feasible solutions – Northwest corner rule, least cost method	1
3.4	Vogel's approximation method. MODI method	2
3.5	Assignment problem, Formulation of assignment problem	1
3.6	Hungerian method for optimal solution, Solution of unbalanced problem. Travelling salesman problem	2
4	**Sequencing and Scheduling - (9 hours)**	
4.1	Introduction, Problem of Sequencing, the problem of n jobs and two machines	2
4.2	problem of m jobs and m machines	1
4.3	Scheduling Project management-Critical path method (CPM)	2
4.4	Project evaluation and review technique (PERT),	2
4.5	Optimum scheduling by CPM, Linear programming model for CPM and PERT.	2
5	**Non Linear Programming - (9 hours)**	
5.1	Examples , Graphical illustration, One variable unconstrained optimization	2
5.2	Multiple variable unconstraint optimization-- gradient search	2
	The Karush –Kuhn Tucker condition for constraint optimization	1
5.3	Quadratic programming-modified simplex method-	2
5.5	Separable programming	2

SEMESTER V

MINOR

CODE	Course Name	CATEGORY	L	T	P	CREDIT
MAT 381	RANDOM PROCESS AND QUEUEING THEORY	VAC	3	1	0	4

Preamble: This course introduces learners to the applications of probability theory in the modelling and analysis of stochastic systems, covering important models of random processes such as Poisson Process, Markov chain and queueing systems. The tools and models introduced here have important applications in engineering and are indispensable tools in signal analysis, reliability theory, network queues and decision analysis.

Prerequisite: A basic knowledge in calculus and matrix algebra**.**

Course Outcomes: After the completion of the course the student will be able to

CO1	Characterize phenomena which evolve probabilistically in time using the tools autocorrelation and power spectrum (Cognitive knowledge level: **Understand**).
CO2	Characterize stationary processes using ergodic property and analyse processes using poisson model wherever appropriate (Cognitive knowledge level: **Apply**).
CO3	Model and analyze random phenomena using discrete time Markov chains (Cognitive knowledge level: **Apply**).
CO4	Explain basic characteristic features of a queuing system and analyse queuing models (Cognitive knowledge level: **Apply**).
CO5	Analyse complex queueing systems by applying basic principles of queueing theory (Cognitive knowledge level: **Apply**).

Mapping of course outcomes with program outcomes

	PO1	**PO2**	**PO3**	**PO4**	**PO5**	**PO6**	**PO7**	**PO8**	**PO9**	**PO10**	**PO11**	**PO12**
CO 1	3	2	2	2								2
CO 2	3	2	2	2								2
CO 3	3	2	2	2								2
CO 4	3	2	2	2						2		2
CO 5	3	2	2	2								2

Abstract POs defined by National Board of Accreditation			
PO#	**Broad PO**	**PO#**	**Broad PO**
PO1	Engineering Knowledge	**PO7**	Environment and Sustainability
PO2	Problem Analysis	**PO8**	Ethics
PO3	Design/Development of solutions	**PO9**	Individual and team work
PO4	Conduct investigations of complex problems	**PO10**	Communication
PO5	Modern tool usage	**PO11**	Project Management and Finance
PO6	The Engineer and Society	**PO12**	Life long learning

Assessment Pattern

Bloom's Category	Continuous Assessment Tests (%)		End Semester Examination (%)
	1	2	
Remember	20	20	20
Understand	35	35	35
Apply	45	45	45
Analyse			
Evaluate			
Create			

Mark Distribution

Total Marks	CIE Marks	ESE Marks	ESE Duration
150	50	100	3 hours

Continuous Internal Evaluation Pattern:

Attendance : 10 marks

Continuous Assessment - Test : 25 marks

Continuous Assessment - Assignment : 15 marks

Internal Examination Pattern:

Each of the two internal examinations has to be conducted out of 50 marks. First series test shall be preferably conducted after completing the first half of the syllabus and the second series test shall be preferably conducted after completing remaining part of the syllabus. There will be two parts: Part A and Part B. Part A contains 5 questions (preferably, 2 questions each from the completed modules and 1 question from the partly completed module), having 3 marks for each question adding up to 15 marks for part A. Students should answer all questions from Part A. Part B contains 7 questions (preferably, 3 questions each from the completed modules and 1 question from the partly completed module), each with 7 marks. Out of the 7 questions, a student should answer any 5.

End Semester Examination Pattern:

There will be two parts; Part A and Part B. Part A contains 10 questions with 2 questions from each module, having 3 marks for each question. Students should answer all questions. Part B contains 2 questions from each module of which a student should answer any one. Each question can have maximum 2 sub-divisions and carries 14 marks.

MAT 381 - RANDOM PROCESS AND QUEUEING THEORY

Syllabus

Module 1 (Random processes and stationarity)

Random processes-definition and classification, mean, autocorrelation, stationarity-strict sense and wide sense, properties of autocorrelation function of WSS processes.

Power spectral density of WSS processes and its properties- relation to autocorrelation function. White noise.

Module 2 (Poisson processes)

Ergodic processes-ergodic in the mean and autocorrelation. Mean ergodic theorems (without proof).

Poisson processes-definition based on independent increments and stationarity, distribution of inter-arrival times, sum of independent Poisson processes, splitting of Poisson processes.

Module 3 (Markov chains)

Discrete time Markov chain -Transition probability matrix, Chapman Kolmogorov theorem (without proof), computation of probability distribution, steady state probabilities. Classification of states of finite state chains , irreducible and ergodic chains.

Module 4 (Queueing theory-I)

Queueing systems, Little's formula (without proof), Steady state probabilities for Poisson queue systems, M/M/1 queues with infinite capacity and finite capacity and their characteristics-expected number of customers in queue and system, average waiting time of a customer in the queue and system

Module 5 (Queueing theory-II)

Multiple server queue models, M/M/s queues with infinite capacity, M/M/s queues with finite capacity-in all cases steady state distributions and system characteristics-expected number of customers in queue and system, average waiting time of a customer in the queue and system

Books

1. Alberto leon Garciai, Probability and random processes for electrical engineering, Pearson Education, Second edition

2. V Sundarapandian, Probability statistics and queueing theory, Prentice-Hall Of India.

Assignments

Assignments should include specific problems highlighting the applications of the methods introduced in this course in physical sciences and engineering.

Sample Course Level Assessment Questions

Course Outcome 1 (CO1):

1. What are the various classes of random processes? Explain with examples
2. Consider the random process $X(t) = a cos(\omega t + \Theta)$ where $a$and $\omega$are constants and Θ is a random variable uniformly distributed in $(0,2\pi)$. Show that $X(t)$is WSS.
3. If $X(t)$ is a wide sense stationary process with autocorrelation function $R_X(\tau) = 3 + 9e^{-3|\tau|}$, find the mean, variance and average power of the process.
4. Given that a random process $x(t)$ has power spectral density $S_X(\omega) = \dfrac{1}{1+\omega^2}$ of a WSS process, find the average power of the process.

Course Outcome 2 (CO2)

1. Give one example each of a process which is (i) ergodic (ii) non-ergodic.
2. Derive the mean, autocorrelation and autocovariance of a Poisson process.
3. Suppose that customers arrive at a bank according to a Poisson process with a mean rate of 20 per hour. Find the probability that during a time interval of 10 minutes (i) exactly 3 customers arrive (ii) more than 3 customers arrive.
4. Prove that the inter-arrival time of a Poisson process follows an exponential distribution.

Course Outcome 3(CO3):

1. Give an example of a discrete time Markov process
2. Consider the experiment of sending a sequence of messages across a communication channel. Due to noise, there is a small probability p that the message may be received in error. Let X_n denote the number of messages received correctly up to and including the n-th transmission. Show that $X_n$is a homogeneous Markov chain. What are the transition probabilities?
3. Find the steady state probability distribution of a Markov chain with transition probability matrix
$$P = \begin{bmatrix} 0.2 & 0.3 & 0.5 \\ 0.1 & 0.6 & 0.3 \\ 0.4 & 0.3 & 0.3 \end{bmatrix}$$
4. Three boys A, B, C are throwing a ball to each other. A always throws the ball to B and B always throws the ball to C, but C is as likely to throw the ball to B as to A. Show that the process is Markovian. Find the transition probability matrix and classify the states.

Course Outcome 4(CO4):

1. What are the basic characteristics of a queueing system
2. Derive the expressions for the steady state probability distribution of a Poisson queueing system
3. A concentrator receives messages from a group of terminals and transmits them over a single transmission line. Suppose that messages arrive according to a Poisson process at a rate of 1 message every 4 milliseconds, and suppose that message transmission times are exponentially distributed with mean 3 ms. Find the mean number of messages in the system and the mean total delay in the system. What percentage increase in the arrival rate results in doubling of the above mean total delay.
4. Patients arrive at a doctor's clinic according to Poisson distribution at a rate of 30 per hour. The waiting room does not accommodate more than 9 patients. Examination time per patient is exponential with a mean rate of 20 per hour. Find the probability that an arriving patient will have to go back because the waiting room is full.

Course Outcome 5 (CO5):

1. Obtain the steady state probability distribution of an M/M/s queueing system with infinite capacity.
2. A company has four printers to handle the print jobs arriving at a server. Suppose that print jobs arrive according to a Poisson process at a rate of one job every 2 minutes, and suppose the printing durations are exponentially distributed with mean 4 minutes. When all printers are busy the system queues the call requests until a line becomes available. Find the probability that a print job will have to wait.
3. How will you model the mean arrival rate and mean service rate in a Poisson queueing system with 4 servers and capacity limited to 5?
4. A dispensary has two doctors and four chairs in the waiting room. The patients who arrive at the dispensary leave if they find all the chairs occupied. Patients arrive at an average rate of 8 per hour and spend an average of 10 minutes for their check-up. The arrival process is assumed to be Poisson and the service times are exponential. Find the probability that an arriving patient will not have to wait. What is the expected waiting time of a patient in the queue?

MODEL QUESTION PAPER

APJ ABDUL KALAM TECHNOLOGICAL UNIVERSITY

Course Code: MAT381

Course Name: Random process and queueing theory

Max. Marks: 100 Duration: 3 Hours

PART A

Answer all questions. Each question carries 3 marks Marks

1	What are the various classes of random processes? Give examples	(3)
2	Consider the random process $X(t) = c$ where c is a constant. Is it SSS? WSS?	(3)
3	Explain the terms mean-ergodic process, correlation ergodic process	(3)
4	Find the autocorrelation of a Poisson process	(3)
5	A fair die is tossed repeatedly and let X_n denote the maximum of the numbers obtained upto the n-th toss. Is X_n a Markov chain? Justify.	(3)
6	Prove that if P is a Markov matrix then P^2 is also a Markov matrix	(3)
7	What do the letters in the symbolic representation (a/b/c): (d/e) of a queueing model represent?	(3)
8	What are the conditions for a M/M/1 queueing system to have a steady state distribution?	(3)
9	Find the probability that an arriving customer is forced to join the queueing system M/M/s.	(3)
10	A two-server queueing system is in a steady state condition and the steady state probabilities are $p_0 = \frac{1}{16}, p_1 = \frac{4}{16}, p_2 = \frac{6}{16}, p_3 = \frac{4}{16}, p_4 = \frac{1}{16}$ and $p_n = 0$ if $n > 4$. Find the mean number of customers in the system and in the the queue.	(3)

PART B

Answer any one full question from each module. Each question carries 14 marks

Module 1

11 (a) Show that the mean of a first order stationary random process is a constant. (7)

(b) Consider the random process $X(t) = Acos(\omega t)$ where $\omega$is a constant and $A$is a random variable uniformly distributed in $(0,2\pi)$. Find the mean and autocorrelation of Is $X(t)$. Is it stationary? Justify. . (7)

12 (a) Find the mean and variance of a WSS process with autocorrelation function $R_X(\tau) = 1 + 4e^{-3|\tau|}$. (7)

(b) Let X(t) and Y(t) be both zero-mean, uncorrelated WSS random processes. Consider the random process Z(t) defined by .Determine the autocorrelation function and the power spectral density ofZ(t) (7)

Module 2

13 (a) Using mean ergodic theorem show that a constant random process $X(t) = C$, where C is a random variable with mean $\mu$and variance σ^2 , is not mean ergodic. (7)

(b) Patients arrive at the doctor's office according to a Poisson process with rate $\lambda = \frac{1}{10}$ minute. The doctor will not see a patient until at least three patients are in the waiting room. Find the expected waiting time until the first patient is admitted to see the doctor. (7)

14	(a) The number of telephone calls arriving at a certain switch board within a time interval of length measured in minutes is a Poisson process with parameter $\lambda = 2$. Find the probability of (i) No telephone calls arriving at this switch board during a 5 minute period. (ii) More than one telephone calls arriving at this switch board during a given $\frac{1}{2}$minute period.	(6)
	(b) Let $X(t)$ be a Poisson process with rate λ . Find (i) $E[X^2(t)]$ (ii) $E\{[X(t) - X(s)]^2\}$ for $t > s$.	(8)

Module 3

15	The transition probability matrix of a Markov chain $\{X_n, n \geq 0.\}$ with three state 1,2 and 3 is $\begin{bmatrix} 0.5 & 0.3 & 0.2 \\ 0.4 & 0.2 & 0.4 \\ 0.1 & 0.6 & 0.3 \end{bmatrix}$ and the initial probability distribution is $p(0) = [0.5\ 0.3\ 0.2]$. FInd (a) $P\{X_2 = 2\}$ (b) $P\{X_3 = 3, X_2 = 2, X_1 = 1, X_0 = 3\}$	(14)
16	Let $\{X_n;\ n = 1,2,3,...\}$be a discrete time Markov Chain with state space $S = \{0,1,2\}$and one step transition probability matrix given by $P = \begin{bmatrix} 0 & 1 & 0 \\ 0.25 & 0.5 & 0.25 \\ 0 & 1 & 0 \end{bmatrix}$ (a) Is the chain ergodic? Explain. (b) Find the invariant probabilities.	(14)

Module 4

17 (a) Find the mean number of customers in the queue, system, average waiting time in the queue and system of M/M/I queueing model with infinite capacity. (8)

(b) A T.V repairman finds that the time spent on his job has an exponential distribution with mean 30 minutes. He repairs the sets in the order in which they came in. The arrival of sets is approximately Poisson with an average rate of 10 per 8 hours a day. (6)

(i) Find the repairman's expected idle time each day.

(ii) Find the average number of jobs he handles on a given day.

18 Customers arrive at a one-window drive-in bank according to a Poisson distribution, with a mean of 10 per hour. The service time per customer is exponential, with a mean of 5 minutes. There are three spaces in front of the window, including the car being served. Other arriving cars line up outside this 3-car space. (14)

(a) What is the probability that an arriving car can enter one of the 3-car spaces?

(b) What is the probability that an arriving car will wait outside the designated 3-car space?

(c) How long is an arriving customer expected to wait before starting service?

(d) How many car spaces should be provided in front of the window (including the car being served) so that an arriving car can find a space there at least 90% of the time?

Module 5

19 A telephone exchange has two long distance operators. It is observed that long distance calls arrive in a Poisson fashion at an average rate of 15 per hour. The length of service on these calls is approximately exponential distributed with mean length 2 minutes. Find (14)

(i) the probability a subscriber will have to wait for a long distance call,

(ii) the expected number of customers in the system,

(iii) the expected number of customers in the queue,

(iv) the expected time a customers spends in the system and

(v) the expected waiting time for a customer in the queue.

20 A dispensary has two doctors and four chairs in the waiting room. The patients who arrive at the dispensary leave when all four chairs in the waiting room of the dispensary are occupied. It is known that the patients arrive at the average rate of 8 per hour and spend an average of 10 minutes for their check-up and medical consultation. The arrival process is Poisson and the service time is an exponential random variable. Find (14)

(i) the probability that an arriving patient will not wait,

(ii) the effective arrival rate at the dispensary,

(iii) the expected number of patients at the queue,

(iv) the expected waiting time of a patient at the queue,

(v) the expected number of patients at the dispensary and

(vi) the expected time a patient spends at the dispensary.

Teaching Plan

No	Topic	No. of Lectures
1	**Random processes and stationarity**	**9 hours**
1.1	Random-process, classification,	1
1.2	Mean, variance, autocorrelation, autocovariance	1
1.3	Strict sense stationary processes	1
1.4	WSS processes (Lecture 1)	1
1.5	WSS processes (Lecture 2)	1
1.6	Properties of autocorrelation of a WSS process	1
1.7	Power spectral density, relation to autocorrelation	2
	Delta function, white noise	1
2	**Ergodicity, Poisson process**	**9 hours**
2.1	Ergodic property, definition, examples	1
2.2	Mean ergodic theorems and applications (Lecture 1)	1
2.3	Mean ergodic theorems and applications (Lecture 2)	1
2.4	Poisson process-independent increments, stationarity (Lecture 1)	1
2.5	Poisson process-independent increments, stationarity (Lecture 2)	1
2.6	Mean, variance, autocorrelation, autocovariance of Poisson process	1
2.7	Distribution of inter-arrival times	1
2.8	Splitting (thinning) of Poisson processes	1
2.9	Merging of Poisson process	1
3	**Discrete time Markov chains**	**9 hours**
3.1	Discrete time Markov chain-memorylessness, examplesition probability matrix, Chapman-Kolmogorov theorem	1
3.2	Transition probabilities and transition matrices	1
3.3	Chapman-Kolmogorov theorem and applications	1
3.4	Computation of transient probabilities (Lecture 1)	1
3.5	Computation of transient probabilities (Lecture 2)	1
3.6	classification of states of finite-state chains, irreducible and ergodic chains (Lecture 1)	1
3.7	classification of states of finite-state chains, irreducible and ergodic chains (Lecture 2)	1
3.8	Steady state probability distribution of ergodic chains (Lecture 1)	1

3.9	Steady state probability distribution of ergodic chains (Lecture 2)	1
4	**Queueing theory 1**	**9 hours**
4.1	Basic elements of Queueing systems, Little's formula,	1
4.2	Steady state probabilities for Poisson queue systems (Lecture 1)	1
4.3	Steady state probabilities for Poisson queue systems (Lecture 2)	1
4.4	M/M/1 queues with infinite capacity, steady state probabilities	1
4.5	M/M/1 queues with infinite capacity- computating system characteristics (Lecture 1)	1
4.6	M/M/1 queues with infinite capacity- computating system characteristics (Lecture 2)	1
4.7	M/M/1 queues with finite capacity, steady state probabilities	1
4.8	M/M/1 queues with finite capacity- computating system characteristics (Lecture 1)	1
4.9	M/M/1 queues with finite capacity- computating system characteristics (Lecture 2)	1
5	**Queueing theory II**	**9 hours**
5.1	Basic elements of multiple server queues	1
5.2	M/M/s queues with infinite capacity, steady state probabilities (Lecture 1)	1
5.3	M/M/s queues with infinite capacity, steady state probabilities (Lecture 2)	1
5.4	M/M/s queues with infinite capacity- computing system characteristics (Lecture 1)	1
5.5	M/M/s queues with infinite capacity- computing system characteristics (Lecture 2)	1
5.6	M/M/s queues with finite capacity, steady state probabilities (Lecture 1)	1
5.7	M/M/s queues with finite capacity, steady state probabilities (Lecture 2)	1
5.8	M/M/s queues with finite capacity- computing system characteristics (Lecture 1)	1
5.9	M/M/s queues with finite capacity- computing system characteristics (Lecture 2)	1

SEMESTER VI

MINOR

CODE	COURSE NAME	CATEGORY	L	T	P	CREDIT
MAT382	ALGEBRA AND NUMBER THEORY	VAC	3	1	0	4

Preamble: This is an introductory course in algebra and number theory with special emphasis on applications including RSA, prime factorization and the interplay between rings and numbers.

Prerequisite: A basic understanding of set theory and logic.

Course Outcomes: After the completion of the course the student will be able to

CO1	Solve number theoretic problems by applying the concept and properties of natural numbers and applications of division algorithm and related results (Cognitive knowledge level: **Apply**).
CO2	Utilise the concepts and properties learned about prime numbers and basic factorisation algorithms to solve number theoretic problems (Cognitive knowledge level: **Apply**).
CO3	Solve algebraic problems using the concepts and properties of groups and group structures (Cognitive knowledge level: **Apply**).
CO4	Utilise the concept, properties and applications of cyclic groups, permutations and symmetric groups to solve algebraic problems (Cognitive knowledge level: **Apply**).
CO5	Solve algebraic problems using the concept, properties and applications of rings and ring structures (Cognitive knowledge level: **Apply**).

Mapping of course outcomes with program outcomes

	PO1	PO2	PO3	PO4	PO5	PO6	PO7	PO8	PO9	PO10	PO11	PO12
CO1	3	2	2	2								2
CO2	3	2	2	2								2
CO3	3	2	2	2								2
CO4	3	2	2	2								2
CO5	3	2	2	2								2

Abstract POs defined by National Board of Accreditation			
PO#	**Broad PO**	**PO#**	**Broad PO**
PO1	Engineering Knowledge	PO7	Environment and Sustainability
PO2	Problem Analysis	PO8	Ethics
PO3	Design/Development of solutions	PO9	Individual and team work
PO4	Conduct investigations of complex problems	PO10	Communication
PO5	Modern tool usage	PO11	Project Management and Finance
PO6	The Engineer and Society	PO12	Life long learning

Assessment Pattern:

Bloom's Category	Continuous Assessment Tests		End Semester Examination
	1	2	
Remember	5	5	10
Understand	10	10	20
Apply	35	35	70
Analyse			
Evaluate			
Create			

Mark Distribution

Total Marks	CIE Marks	ESE Marks	ESE Duration
150	50	100	3 hours

Continuous Internal Evaluation Pattern:

Attendance : 10 marks

Continuous Assessment - Test : 25 marks

Continuous Assessment - Assignment : 15 marks

Internal Examination Pattern:

Each of the two internal examinations has to be conducted out of 50 marks. First series test shall be preferably conducted after completing the first half of the syllabus and the second series test shall be preferably conducted after completing remaining part of the syllabus. There will be two parts: Part A and Part B. Part A contains 5 questions (preferably, 2 questions each from the completed modules and 1 question from the partly completed module), having 3 marks for each question adding up to 15 marks for part A. Students should answer all questions from Part A. Part B contains 7 questions (preferably, 3 questions each from the completed modules and 1 question from the partly completed module), each with 7 marks. Out of the 7 questions, a student should answer any 5.

End Semester Examination Pattern:

There will be two parts; Part A and Part B. Part A contains 10 questions with 2 questions from each module, having 3 marks for each question. Students should answer all questions. Part B contains 2 questions from each module of which a student should answer any one. Each question can have maximum 2 sub-divisions and carries 14 marks.

Syllabus

Module 1 (Elementary Number Theory)

Division with remainder, congruences, greatest common divisor, Euclidean algorithm, Chinese remainder theorem, Euler's theorem **(**Sections 1.2-1.7)

Module 2 (Prime Numbers)

Prime Numbers- basic results, unique factorisation, computing Euler $\varphi-$ function, RSA explained, Fermat's little theorem, pseudoprimes, Algorithms for prime factorisation- Fermat's and Fermat-Kraitchik algorithms (evaluation only), Quadratic residues. **(**Relevant topics from sections 1.8-1.11)

Module 3 (Introduction to Groups)

Groups- Definition- basic properties and examples, subgroups and cosets, normal subgroups, group homomorphisms. Isomorphism theorem (Sections 2.1- 2.5)

Module 4 (Further topics in Group theory)

Order of a group element, Cyclic groups, symmetric groups, cycles, simple transpositions and bubble sort, alternating groups. (Sections 2.6-2.7, 2.9.1, 2.9.2, 2.9.3)

Module 5 (Ring Theory)

Rings- Definition, ideals, principal ideal domain, Quotient rings, Prime and maximal ideals, Ring homomorphisms, unique factorisation domain, irreducible and prime elements, Euclidean domain. (Sections 3.1, 3.2, 3.3, 3.3.1, 3.5.1-3.5.4)

Text Book

Niels Lauritzen, "Concrete Abstract Algebra", Cambridge University Press,2003

Reference Books

1. David M Burton, "Elementary Number Theory", 7th edition, McGraw Hill, 2011
2. John B Fraleigh, "A first course in Abstract Algebra". 7th edition, Pearson Education India, 2013
3. Joseph A Gallian, "Contemporary Abstract Algebra", 9th edition, Cengage Learning India Pvt. Ltd

Assignments

Assignments should include specific problems highlighting the applications of the methods introduced in this course in physical sciences and engineering.

Sample Course Level Assessment Questions

Course Outcome 1 (CO1):

1. Find the remainder of 2^{340} after division by 341 using repeated squaring algorithm.
2. What is the smallest natural number that leaves a remainder of 2 when divided by 3 and a remainder of 3 when divided by 5 ?

Course Outcome 2 (CO2)

1. Find a prime factorization of 2041 using Fermat Kraitchik algorithm.
2. What is the product of the greatest common divisor and least common multiple of 2 numbers ?

Course Outcome 3(CO3):

1. Write down the subgroups of Z/8Z.
2. Show that every subgroup of an abelian group is normal.

Course Outcome 4(CO4):

1. Prove that $(Z/13Z)^*$ is a cyclic group.
2. Write $\sigma = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 6 & 5 & 4 & 3 & 2 & 1 \end{pmatrix} \in S_6$ as a product of the minimal number of simple transpositions.

Course Outcome 5 (CO5):

1. Write down the units of Z/8Z .
2. Show that $Z\left[\sqrt{-6}\,\right]$ is not a Unique Factorisation Domain .

Model Question Paper

APJ ABDULKALAM TECHNOLOGICAL UNIVERSITY

SIXTH SEMESTER B.TECH. DEGREE

EXAMINATION (MONTH & YEAR)

Course Code: MAT382

Course Name: ALGEBRA AND NUMBER THEORY

MAX.MARKS: 100 DURATION: 3 Hours

PART A

Answer all questions, each question carries 3 marks.

1. Find the remainder when 2^{50} is divided by 7.
2. Prove that if $a|bc$ with $gcd(a,b) = 1$, then $a|c$.
3. Prove that there exists infinitely many prime numbers.
4. Prove that 25 is a strong pseudoprime relative to 7.
5. Prove that a group has only one idempotent element.
6. Find all the subgroups of $\mathbb{Z}/6\mathbb{Z}$.
7. Write down all the elements of order 7 in $\mathbb{Z}/28\mathbb{Z}$.
8. Find the generators of $\mathbb{Z}_{18}$.
9. Find a zero divisor in $\mathbb{Z}_5[i] = \{a + ib: a, b \in \mathbb{Z}_5\}$.
10. Write down all the maximal ideals in $\mathbb{Z}_{10}$.

PART B

Answer any one full question from each module, each question carries 14 marks.

Module-I

11. (a) Compute $\lambda, \mu \in \mathbb{Z}$ such that $89\lambda + 55\mu = 1$ and find all solutions $x \in \mathbb{Z}$ to $89x \cong 7mod(55)$.

(b) Solve the system of simultaneous congruences $x \cong 2(mod3)$, $x \cong 3(mod5)$, $x \cong 2(mod7)$.

12. (a) Suppose $a, b \in \mathbb{N}$ such that $gcd(a, b) = 1$. Prove that $gcd(a^m, b^n) = 1$, for $m, n \in \mathbb{N}$

(b) Use Euclidean algorithm to find integers x and y satisfying

$gcd(1769,2378) = 1769x + 2378y$

Module – II

13. (a) Using Fermat's factorization method factorise $2^{11} - 1$.

(b) Decrypt the cipher text 1030 1511 0744 1237 1719 that was encrypted using the RSA algorithm using the public key $(N, e) = (2623,869)$.

14. (a) Determine the quadratic residues and non-residues modulo 13.

(b) Show that $\varphi(n) = \varphi(2n)$, if n is odd.

Module-III

15. (a) Prove that $GL_2(\mathbb{R})$ is a non abelian group.

(b) Let $\varnothing\colon\ S_n \to \mathbb{Z}_2$ defined by $\varnothing(\ \sigma) = \begin{cases} 0, \ if\ \sigma \text{ is even} \\ 1, \ if\ \sigma \text{ is odd} \end{cases}$. Prove that $\varnothing$ is a homomorphism.

Also find Ker $\varnothing$.

16. (a) Show that every subgroup of an abelian group is normal.

(b) Let $\varnothing\colon\ G \to G'$ where G and G' are groups. Prove that $\varnothing$ is an isomorphism if and only if Ker $\varnothing = \{e\}$.

Module-IV

17. (a) Prove that an even permutation cannot be the product of an odd number of transpositions

(b) Show that every permutation $\sigma \in S_n$ is a product of unique disjoint cycles.

18. (a) Show that if σ is a cycle of odd length then σ^2 is a cycle.

(b) Check whether $(\mathbb{Q}\backslash\{0\},\ .)$ is a cyclic group.

Module- V

19. (a) Show that every field is a domain. Is the converse of the statement true ? Justify.

(b) Write all the units of the Gaussian integers $\mathbb{Z}$ [i].

20. (a) Prove that every principal ideal domain is a unique factorisation domain.

(b) Let R be a non-commutative ring. Prove that R/P is a domain if P is a prime ideal.

Teaching Plan		
Sl. No	**Topic**	**No. Of Lecture Hours**
1	**Elementary Number Theory**	**8 Hours**
1.1	Division with remainder	1
1.2	Congruence	1
1.3	Properties of Congruence	1
1.4	Greatest Common divisor	1
1.5	Euclidean algorithm	1
1.6	Relatively prime numbers	1
1.7	Chinese Remainder Theorem	1
1.8	Euler's Theorem	1
2	**Prime Numbers**	**9 Hours**
2.1	Basic Results	1
2.2	unique factorisation	1
2.3	Computing $\varphi-$function	1
2.4	RSA explained	1
2.5	Fermat's Little theorem, Pseudoprimes	1
2.6	Factorisation algorithms- Fermat's algorithm	1
2.7	Fermat-Kraitchik algorithm	1
2.8	Quadratic residue	1
2.9	Quadratic residue applications	1
3	**Introduction to Groups**	**9 Hours**
3.1	Definition	1
3.2	Basic Properties	1
3.3	Examples	1
3.4	Subgroups	1
3.5	Cosets	1

3.6	Normal Subgroups	1
3.7	Quotient Groups	1
3.8	Group homomorphisms	1
3.9	Isomorphism theorem	1
4	**Further topics in Group Theory**	**9 Hours**
4.1	Order of a group element	1
4.2	Cyclic Groups	1
4.3	Properties	1
4.4	Symmetric groups	1
4.5	Cycles	1
4.6	Properties	1
4.7	Simple transpositions	1
4.8	Bubble sort	1
4.9	Alternating groups	1
5	**Ring Theory**	**9 Hours**
5.1	Definition, basic properties,	1
5.2	ideals	1
5.3	Quotient rings	1
5.4	Prime and Maximal ideals	1
5.5	Ring homomorphisms,	1
5.6	Unique factorisation	1
5.7	Irreducible elements	1
5.8	prime elements	1
5.9	Euclidean domain	1

SEMESTER VII

MINOR

<table>
<tr><td rowspan="2">MAT481</td><td rowspan="2">FUNCTIONAL ANALYSIS</td><td>Category</td><td>L</td><td>T</td><td>P</td><td>CREDIT</td><td>YEAR OF INTRODUCTION</td></tr>
<tr><td>VAC</td><td>3</td><td>1</td><td>0</td><td>4</td><td>2019</td></tr>
</table>

Preamble: This course will cover the foundations of functional analysis in the context of basic real analysis, Metric spaces, Banach spaces and Hilbert spaces. Students learn various types of distances and associated results in these spaces. The important notion of linear functionals and duality will be developed in Banach space. An introduction to the concept of orthonormal sequences in Hilbert spaces enables them to efficiently handle with a variety of applications in engineering problems.

Prerequisite: Basic knowledge in set theory and linear algebra

Course Outcomes: After the completion of the course the student will be able to

CO1	Explain the concept and analytical properties of the real number system (Cognitive knowledge level: **Understand**).
CO2	Illustrate the concept of metric space and discuss the properties interior, closure, denseness and separability in a metric space (Cognitive knowledge level: **Understand**).
CO3	Explain the concepts of Cauchy sequence, completeness and Banach spaces and apply these concepts to metric and Banach spaces (Cognitive knowledge level: **Apply**).
CO4	Demonstrate the concepts of linear operator, linear functional, dual basis and dual space of normed linear spaces (Cognitive knowledge level: **Understand**).
CO5	Explain the notions of inner product and Hilbert space and apply the tools to construct orthonormal sequences in Hilbert spaces (Cognitive knowledge level: **Apply**).

Mapping of course outcomes with program outcomes

	PO1	PO2	PO3	PO4	PO5	PO6	PO7	PO8	PO9	PO10	PO11	PO12
CO1	3			2					1	2		2
CO2	3	3	2	2					1	2		2
CO3	3	3	2	2					1	2		2
CO4	3	3	2	2					1	2		2
CO5	3	3	2	2					1	2		2

Abstract POs defined by National Board of Accreditation			
PO#	**Broad PO**	**PO#**	**Broad PO**
PO1	Engineering Knowledge	PO7	Environment and Sustainability
PO2	Problem Analysis	PO8	Ethics
PO3	Design/Development of solutions	PO9	Individual and team work
PO4	Conduct investigations of complex problems	PO10	Communication
PO5	Modern tool usage	PO11	Project Management and Finance
PO6	The Engineer and Society	PO12	Life long learning

Assessment Pattern

Bloom's Category	Continuous Assessment Tests(%)		End Semester Examination(%)
	1	2	
Remember	20	20	20
Understand	30	30	30
Apply	50	50	50
Analyse			
Evaluate			
Create			

Continuous Internal Evaluation Pattern:

Attendance : 10 marks

Continuous Assessment - Test : 25 marks

Continuous Assessment - Assignment : 15 marks

Internal Examination Pattern:

Each of the two internal examinations has to be conducted out of 50 marks. First series test shall be preferably conducted after completing the first half of the syllabus and the second series test shall be preferably conducted after completing remaining part of the syllabus. There will be two parts: Part A and Part B. Part A contains 5 questions (preferably, 2 questions each from the completed modules and 1 question from the partly completed module), having 3 marks for each question adding up to 15 marks for part A. Students should answer all questions from Part A. Part B contains 7 questions (preferably, 3 questions each from the completed modules and 1 question from the partly completed module), each with 7 marks. Out of the 7 questions, a student should answer any 5.

End Semester Examination Pattern:

There will be two parts; Part A and Part B. Part A contains 10 questions with 2 questions from each module, having 3 marks for each question. Students should answer all questions. Part B contains 2 questions from each module of which a student should answer any one. Each question can have maximum 2 sub-divisions and carries 14 marks.

Syllabus

Module 1 (Real Analysis) (9 hours)

(Text 1 - Relevant topics from sections 1.3, 2.3, 3.1, 3.2, 3.3, 3.4, 3.5, 8.1)

Denumerable set, Countable set, Supremum and Infimum of a set, Sequence of real numbers, Convergent and Divergent sequence, Limit, Bounded sequence, Monotone sequence, Monotone convergence Theorem(without proof), Subsequence, Bolzano-Weierstrass theorem(without proof) , Cauchy sequence, Cauchy convergence criterion, Sequence of functions, Pointwise convergence, Uniform convergence, Uniform norm

Module 2 (Metric Space) (9 hours)

(Text 2 - Relevant topics from sections 1.1, 1.2, 1.3, 1.4[1.4-1 to 1.4- 2])

Metric Space: $\mathbb{R}^n, \mathbb{C}^n,\ l^\infty,\ C[a,b]$, Discrete space, Sequence space, B(A), l^p, Subspace, Holder inequality (without proof), Cauchy- Schwarz inequality (without proof), Minkowski inequality (without proof), Open set, Closed set, Neighbourhood, Interior, Continuous function, Accumulation point, Closure, Dense set, Separable space, Convergence of sequence, Limit, Bounded sequence.

Module 3 (Complete Metric Space and Normed space) (8 hours)

(Text 2 - Relevant topics from sections 1.4[1.4-3 to 1.4-8], 1.5, 2.1, 2.2)

Cauchy sequence in a metric space, Complete Metric Space, Completeness of $\mathbb{R}^n, \mathbb{C}^n,\ l^\infty,\ C[a,b]$, Convergent Sequence space, l^p, Examples of incomplete metric spaces, Vector space with examples, Normed space, Banach space: $\mathbb{R}^n, \mathbb{C}^n,\ l^\infty,\ C[a,b]$, Metric induced by norm, Examples of incomplete normed spaces

Module 4 (Space of Functionals and Operators) (9 hours)

(Text 2 - Relevant topics from sections 2.3, 2.6, 2.7, 2.8, 2.9, 2.10)

Properties of Normed Spaces ,Subspaces , Closed subspace, Schauder basis, Linear Operator, Range , Null space, Bounded Linear Operator, Norm of an operator, Linear operator on a finite dimensional space, Continuous linear operator, Relation between bounded and continuous operators, Linear functional, bounded linear functional, Algebraic dual space, Dual basis, Space B(X,Y) , Completeness of B(X,Y)(without proof), Dual space X' , Examples of dual space

Module - 5 (Hilbert Spaces) (10 hours)

(Text 2 - Relevant topics from sections 3.1, 3.2, 3.3, 3.4)

Inner Product Space, Hilbert Space, Parallelogram equality, Orthogonality, Examples of Hilbert Spaces – $\mathbb{R}^n$, $\mathbb{C}^n$, l^2, Examples of Non-Hilbert spaces - l^p with $p \neq 2$, $C[a,b]$, Polarization identity, Further properties of inner product spaces - Schwartz inequality, Triangle inequality, Continuity of inner product, Subspace of an inner product space and Hilbert Space, Subspace Theorem, Convex set, Minimizing vector Theorem (without proof), Orthogonality Lemma (without proof), Direct sum, Orthogonal complement, Direct sum Theorem, Orthogonal projection, Null space Lemma, Closed subspace Lemma, Dense set Lemma, Orthonormal sets and sequences, Examples and properties, Bessel inequality, Gram-Schmidt process (without proof).

Text Book

1. Robert G. Bartle and Donald R. Sherbert, Introduction to Real Analysis, JohnWiley & Sons, Inc., 4th Edition, 2011.
2. Erwin Kreyszig, Introductory Functional Analysis with Applications, John Wiley & Sons (Asia) Pte Ltd.

Reference Books

1. Ajit Kumar, S. Kumaresan, A Basic Course in Real Analysis, CRC Press.
2. Herbert S. Gaskill, P P Narayanaswami, Elements of Real Analysis, Pearson.
3. Hiroyuki Shima, Functional Analysis for Physics and Engineering – An introduction, CRC Press, Taylor & Francis Group.
4. Balmohan V. Limaye, Linear Functional Analysis for Scientists and Engineers, Springer, Singapore, 2016.
5. Rabindranath Sen, A First Course in Functional Analysis- Theory and Applications, Anthem Press - An imprint of Wimbledon Publishing Company.
6. M. Tamban Nair, Functional Analysis- A first course, Prentice Hall of India Pvt. Ltd.

Assignments

Assignments should include specific problems highlighting the applications of the methods introduced in this course in science and engineering.

Course Level Assessment Questions

Course Outcome 1 (CO1)

1. Show that set of odd numbers greater than 10 is countable by finding a bijection
2. Show that $\lim_{n\to\infty}\left(\frac{2n}{n+1}\right) = 2$, by using the definition$[\epsilon - K(\epsilon)]$ of limit
3. State Bolzano-Weierstrass theorem

Course Outcome 2 (CO2)

1. Let $X = \mathbb{R}^2, x = (x_1, x_2), y = (y_1, y_2) \in X$. Define $d(x, y) =\mid x_1 - y_1 \mid$. Check whether d is a metric on X?
2. Show that $A = \left\{(x, y) \in \mathbb{R}^2/x^2 + y^2 < 2y\right\}$ is open in $\mathbb{R}^2$ under the Euclidean metric.
3. Suppose $f : X \longrightarrow Y$ is a constant function between metric spaces, say $f(x) = y_0$ for all $x \in X$. Show that f is continuous.

Course Outcome 3 (CO3)

1. Show that l^∞ is a complete metric space
2. Let X be the set of all integers and $d(x, y) = |x - y|$. Show that (X, d) is a complete metric space.
3. Prove that $C[a, b]$ is vector space

Course Outcome 4 (CO4)

1. If T is a linear operator, then show that range $R(T)$ is a vector space
2. Find the dual basis of the basis $\{(1, 0, 0), (0, 1, 0), (0, 0, 1)\}$ for $\mathbb{R}^3$
3. Show that $f(x) = \sup_{t\in J} x(t)$, where $J = [a, b]$ defines a linear functional on $C[a, b]$. Does it bounded?

Course Outcome 5 (CO5)

1. Show that every inner product space V is a normed space.
2. If x, y are two elements in a Hilbert space with $\| x \| = 2, \| y \| = 3$ and $\| x + y \| = 5$, then find the value of $\| x - y \|$?
3. Construct an orthonormal sequence of vectors $\{e_1, e_2, e_3\}$ in the Hilbert space $\mathbb{R}^3$ using the sequence of vectors $\{x_1, x_2, x_3\}$ where $x_1 = (1, 1, 1), x_2 = (0, 1, 1), x_3 = (0, 0, 1)$

Model Question Paper

No. of Pages: QP CODE

Reg No:_______________ Name:_______________

APJ ABDUL KALAM TECHNOLOGICAL UNIVERSITY

SEVENTH SEMESTER B.TECH (MINOR) DEGREE EXAMINATION, MONTH & YEAR

Course Code: MAT481

Course Name: Functional Analysis

Max. Marks: 100 **Duration: 3 hours**

PART A

Answer all Questions. Each question carries 3 Marks

1. Let $S = \left\{ 1 - \frac{1}{n} \ / \ n \in \mathbb{N} \right\}$. Find infimum and supremum of S.
2. Show that a sequence in $\mathbb{R}$ can have at most one limit
3. Does $d(x, y) = |x - y|$ define a metric on $\mathbb{R}$? Justify.
4. Show that $A^\circ = A$, for any subset A of a discrete metric space (X, d). [A° : Interior of A]
5. Define metric induced norm and give an example
6. Show that $C[a, b]$ is a vector space
7. If T is a linear operator, then show that null space $N(T)$ is a vector space
8. Find a Schauder basis for the normed space l^2. Justify
9. Let V be the vector space of polynomials with inner product defined by $\langle f, g\rangle = \int_0^1 f(t)g(t)\, dt$, where $f(t) = t + 2$, $g(t) = 3t - 2$. Find $\|f - g\|$.
10. Define orthogonal complement $Y^\perp$ in a Hilbert space H. Also show that $Y^\perp$ is a subspace of H

PART B

Answer any one full question from each module

Module-1

11. (a) Show that convergent sequence of real numbers is bounded (7)

    (b) Check whether the sequence $(x^2 e^{-nx})$ converges uniformly on $[0, \infty)$. Justify (7)

OR

12. (a) Show that the set $\mathbb{Q}$ of all rational numbers is denumerable (7)

    (b) Prove that a sequence (f_n) of bounded functions on $A \subseteq \mathbb{R}$ converges uniformly on A to f if and only if $sup\{|f_n(x) - f(x)|/x \in A\} \longrightarrow 0$ (7)

Module-2

13. (a) If (X, d) is any metric space, show that $d_1 = \frac{d(x,y)}{1+d(x,y)}$ is also a metric on X. (7)

(b) Let (X, d) be a metric space and A, B be subsets of X. Show that $\overline{A \cup B} = \overline{A} \cup \overline{B}$ (7) [$\overline{A}$:Closure of A]

OR

14. (a) Show that the space ℓ^p with $1 \leq p < +\infty$ is separable. (7)

(b) Let $X = (X, d)$ be a metric space. If $x_n \longrightarrow x$ and $y_n \longrightarrow y$, then show that $d(x_n, y_n) \longrightarrow d(x, y)$ (7)

Module-3

15. (a) Let X be the set of all integers and $d(x, y) = |x - y|$. Find the general form of a Cauchy sequence in the metric space (X, d) (9)

(b) Give an example of a incomplete normed space. Justify (5)

OR

16. (a) Show that the space l^∞ is Banach space (9)

(b) Show that every convergent sequence in a metric space is a Cauchy sequence (5)

Module-4

17. (a) If X is the space of ordered n tuples of real numbers and $||x|| = \max_j |\xi_j|$, where $x = (\xi_1, \xi_2, \cdots, \xi_n)$. What is the corresponding norm on the dual space X' (7)

(b) Show that the operator $T : l^\infty \longrightarrow l^\infty$ defined by $T(x) = (\eta_j)$, $\eta_j = \dfrac{\xi_j}{j}$, $x = (\xi_j)$ is a bounded linear operator (7)

OR

18. (a) Show that every linear operator on a finite dimensional normed space X is bounded (7)

(b) Find the norm of operator $T : l^2 \longrightarrow l^2$ defined by $T(x) = \left(\dfrac{\xi_j}{j}\right)$ for each $x = (\xi_j)$ (7)

Module-5

19. (a) Prove that the space ℓ^2 is a Hilbert space with inner product defined by $\langle x, y\rangle = \sum_{j=1}^{\infty} \xi_j \overline{\eta_j}$ (7)

(b) If Y is a finite dimensional subspace of a Hilbert space H, then show that Y is complete. (7)

OR

20. (a) Show that a subspace Y of a Hilbert space H is closed in H if and only if $Y = Y^{\perp\perp}$(7)

(b) Let $x_1(t) = t^2, x_2(t) = t, x_3(t) = 1$. Orthonormalize x_1, x_2, x_3 in this order, on the interval $[-1, 1]$ with respect to the inner product $\langle x, y\rangle = \displaystyle\int_{-1}^{1} x(t)y(t)\, dt$ (7)

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Teaching Plan

No	Topic	No. of Lectures
1	**Real Analysis (9 hours)**	
1.1	Denumerable set ,Countable set, Supremum and Infimum of a set	1
1.2	Sequence of real numbers, Convergent sequence	1
1.3	Limit , Divergent sequence	1
1.4	Bounded sequence, Monotone sequence, Monotone convergence Theorem(without proof)	1
1.5	Subsequence, Bolzano-Weierstrass theorem(without proof)	1
1.6	Cauchy sequence, Cauchy convergence criterion	1
1.7	Sequence of functions, Pointwise convergence	1
1.8	Uniform convergence	1
1.9	Uniform norm	1
2	**Metric Space (9 hours)**	
2.1	Metric Space: $\mathbb{R}^n, \mathbb{C}^n$, l^∞	1
2.2	$C[a,b]$, Discrete space, Sequence space	1
2.3	Space of bounded functions $-$B(A), l^p, Subspace, Holder inequality (without proof), Cauchy- Schwarz inequality (without proof), Minkowski inequality (without proof)	1
2.4	Open set, Closed set	1
2.5	Neighbourhood, Interior	1
2.6	Continuous function, Accumulation point	1
2.7	Closure, Dense set	1
2.8	Separable space	1
2.9	Convergence of sequence, Limit, Bounded sequence	1
3	**Complete Metric Space and Normed space (8 hours)**	
3.1	Cauchy sequence, Complete Metric Space	1
3.2	Completeness of $\mathbb{R}^n, \mathbb{C}^n$	1
3.3	Completeness of l^∞, $C[a,b]$	1

3.4	Completeness of Convergent Sequence space, l^p	1
3.5	Examples of incomplete metric spaces	1
3.6	Vector space with examples, Normed space	1
3.7	Banach space: $\mathbb{R}^n, \mathbb{C}^n, l^\infty, C[a,b]$	1
3.8	Metric induced by norm, Examples of incomplete normed spaces	1
4	**Space of Functionals and Operators (9 hours)**	
4.1	Properties of Normed Spaces ,Subspaces	1
4.2	Closed subspace, Schauder basis	1
4.3	Linear Operator, Range , Null space	1
4.4	Bounded Linear Operator, Norm of an operator, Linear operator on a finite dimensional space	1
4.5	Continuous linear operator, Relation between bounded and continuous operators	1
4.6	Linear functional, bounded linear functional	1
4.7	Algebraic dual space, Dual basis	1
4.8	Space B(X,Y) , Completeness of B(X,Y)(without proof)	1
4.9	Dual space X' , Examples of dual space	1
5	**Hilbert Space (10 hours)**	
5.1	Inner Product Space, Hilbert Space, Parallelogram equality, Orthogonality	1
5.2	Examples of Hilbert Spaces: $\mathbb{R}^n$, $\mathbb{C}^n$, l^2	1
5.3	Examples of Non-Hilbert spaces- l^p with $p \neq 2$, $C[a,b]$, Polarization identity.	1
5.4	Schwartz inequality, Triangle inequality, Continuity of Inner product.	1
5.5	Subspace of an inner product space and Hilbert Space, Subspace Theorem.	1
5.6	Convex set, Minimizing vector Theorem (without proof), Orthogonality Lemma (without proof), Direct sum, Orthogonal complement.	1
5.7	Direct Sum Theorem, Orthogonal Projection, Null space Lemma	1
5.8	Closed subspace Lemma, Dense set Lemma, Orthonormal sets and sequences	1
5.9	Examples and properties of Orthonormal sets, Bessel inequality.	1
5.10	Gram-Schmidt process (without proof).	1